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Bohm diffusion equation in quadrupole

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Abstract : Bohm diffusion equation is formulated for quadrupole geometry The equation is solved for a steady state plasma, and for the edge of plasma The solution was for two cases of Bohm diffusion CD_B = T/16 B and D_B = CT/B) The comparison with the experimental results shows a better agreement when instability factor (C) is taken into account

Keywords Plasma diffusion, Bohm diffusion, plasma losses

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1. Introduction

Anomalous diffusion has been recognized in quadrupole, and attributed to Bohm diffusion [1,2]. Outside the critical surface, MHD instability may play a serious role in this diffusion.

However, diffusion equation in multipoles (main octupole) are presented in many forms [3-5]. Hobbs and Taylor [6] took the quadrupole as an example for their application on diffusion equations.

Since the diffusion equation depends on the total flux $\Gamma(\psi, t)$ of the ions normal to the magnetic surface (ψ), all the previous work have been done by a closed integration around a closed field line of length l. According to Hobbs and Taylor [6], the total flux is :

$$\Gamma(\psi,t) = -D(\psi,t)\frac{\partial n}{\partial \psi}, \tag{1}$$

where n is the density and $D(\psi, t)$ is the surface diffusion coefficient. $D(\psi, t)$ is defined by

$$D(\psi,t) = \oint D\,\boldsymbol{B}\,dl, \tag{2}$$

where D is the diffusion coefficient and $\boldsymbol{B}$ is the magnetic field. But $\Gamma(\psi, t)$ are certainly not constant along the field lines, and it is always necessary to average around a field line to obtain the net flux.

In this attempt, we are going to formulate Bohm diffusion equation for a steady state linear quadrupole. Then we try to solve it for the region of MHD instability. A comparison with the experimental result will be followed.

2. The diffusion equation

In attributing the flux of ions (normal to the magnetic surface ψ) to the density gradient in one dimensional space (x),

$$\Gamma(x,t) = D_B \underset{\rightharpoondown}{\nabla}_x n \equiv \Gamma_x . \tag{3}$$

Since the flux of the particles of velocity v is

$$\Gamma_x = nv \tag{4}$$

in quadrupole space (considering magnetic surface where $B_y = \partial\psi / \partial x$) eq. (3) becomes

$$\Gamma(\psi,t) = n\frac{\partial\psi}{\partial t} = B\Gamma_x ,$$

$$B\Gamma(x,t) \equiv \Gamma(\psi,t) \tag{5}$$

Eq. (5) is the transformation equation between ordinary cartesian coordinate and magnetic surface coordinate. Then the transformed form of eq. (3) is

$$\Gamma(\psi,t) = B^2 \, D_B \, \nabla_\psi n \equiv \Gamma_\psi , \tag{6}$$

where Γ_ψ is the flux in quadrupole space. To get the total flux around a certain closed field line, we have to calculate the average as mentioned before, so

$$\langle \Gamma(\psi,t) \rangle = \frac{\oint \Gamma(\psi,t)\, dl / B}{\oint dl / B}$$

$$= \frac{1}{U}\frac{\partial n}{\partial\psi}\oint B D_B \, dl , \tag{7}$$

where $U = \oint dl / B$, and $\partial n / \partial\psi$ is constant along the field line. The difference is quite obvious between eq. (1) and eq. (7).

If we consider the form of D_B as proposed by Sanduk [7] as

$$D_B = CT / 16B, \tag{8}$$

where C is an instability factor, and take the closed integration ($\oint dl = L$) for eq. (7), we get

$$\langle \Gamma(\psi,t) \rangle = \frac{L}{U} CT \frac{\partial n}{\partial\psi} , \tag{9}$$

where T and C are constants along the field line.

The divergency of the flux [7] becomes

$$\nabla_{\psi} \langle \Gamma(\psi,t) \rangle = \frac{\partial}{\partial \psi}\left(\frac{L}{U} CT \frac{\partial n}{\partial \psi} \right). \tag{10}$$

With the aid of continuity equation, we can get the diffusion equation as

$$\frac{\partial n}{\partial t} = \frac{\partial}{\partial \psi}\left(\frac{L}{U} CT \frac{\partial n}{\partial \psi} \right), \tag{11}$$

where L, U, C, T, and $\partial n / \partial t$ are the functions of ψ.

2.1 Steady state diffusion equation :

The experimental result which we are going to compare with, is for UMIST quadrupole which is a linear and steady state system. Owing to the continuous losses and injection of plasma (in steady state), we add a source term to eq. (11) [8] which becomes

$$\frac{\partial n}{\partial t} - \frac{\partial}{\partial \psi}\left(\frac{L}{U} CT \frac{\partial n}{\partial \psi} \right) = Q(\psi). \tag{12}$$

Now, since $\partial n / \partial t = 0$ and there is an offsetting of losses, we can consider radial flux coming from $\psi = 0.1$ [9] as a source. But MHD theory predicts that the instability growing up outside the critical surface (ψ_c). This position is for minimum U or $\partial U / \partial \psi = 0$. According to quadrupole geometry and for UMIST system, this position is $\psi_c = 0.344$ [10]. So the anomalous flux corresponds to $\psi = \psi_c = 0.344$ and ψ_c will be considered as a source position of this type of radial flux ; then eq. (12) can take the form

$$\frac{\partial}{\partial \psi}\left(\frac{L}{U} CT \frac{\partial n}{\partial \psi} \right) = -Q\,\delta(\psi_c). \tag{13}$$

Except at $\psi = \psi_c$, the density must satisfy $[\delta(\psi_c < \psi) = 0]$,

$$\frac{\partial}{\partial \psi}\left(\frac{L}{U} CT \frac{\partial n}{\partial \psi} \right) = 0. \tag{14}$$

3. The solution

The region of integration is extended from $\psi = \psi_c$ to $\psi = \psi_w = 1.755$, where ψ_w is the position of the wall of vacuum vessel. The boundary conditions are :

$$n(\psi_w) = 0, \;\; p(\psi_c) = P_c \;\text{ and} \tag{15}$$

where $p(\psi) = \partial n / \partial \psi$.

First, let $C = 1/16$ as in ordinary form of D_B. Eq. (14) is one dimensional diffusion equation, and its solution is

$$n = n_c \left[1 - \frac{\int_{\psi c}^{\psi} \frac{1}{<B>T} \partial \psi}{\int_{\psi_c}^{\psi_w} \frac{1}{<B>T} \partial \psi} \right], \tag{16}$$

where n_c is

$$n_c = \frac{P_c L_c T_c}{U_c} \int_{\psi_c}^{\psi_w} \frac{1}{<B>T} \partial\psi .$$

The subscript c refer to the values at the critical surface.

Secondly, if C is considered as a function of ψ [7], the solution takes the form

$$n = n_c \left[1 - \frac{\int_{\psi_c}^{\psi} \frac{1}{<B><C>T} \partial\psi}{\int_{\psi_c}^{\psi_w} \frac{1}{<B><C>T} \partial\psi} \right], \tag{17}$$

where

$$n_c = \frac{P_c L_c T_c C_c}{U_c} \int_{\psi_c}^{\psi_w} \frac{1}{<B>T<C>} \partial\psi .$$

Here, <C> is the average along a closed filed line.

The solution of equation of the type proposed by Hobbs and Taylor [6] is also examined and its solution is

$$n = n_c \left[1 - \frac{\int_{\psi_c}^{\psi} \frac{1}{TL<C>} \partial\psi}{\int_{\psi_c}^{\psi_w} \frac{1}{TL<C>} \partial\psi} \right]. \tag{18}$$

The parameters <*B*> and L are calculated numerically and are shown in Figure 1. The experimental data has been extracted by an OML probe which is T-shaped and is made of molybdenum, (cylindrical of 5×10^{-5} m radius), where Debye length is of the order of 10^{-4}m [2].

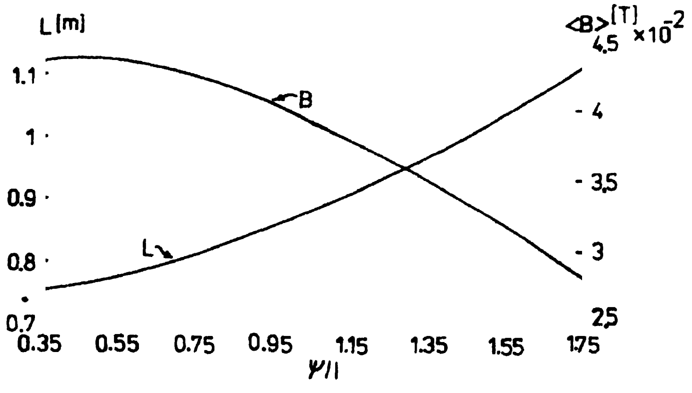

Figure 1. Average field (<*B*>) and length of field line (L) as functions of ψ/I

The temperature T is determined experimentally with in an error range ± 0.02 eV. The factor <*C*> is estimated by using eq. (14) in ref. [7], where all its parameters are known either numerically or experimentally. Figure 2 shows T and the factor <*C*> as functions of ψ/I.

Since $<B>$, L T and $<C>$ are continuous finite real functions with a single real parameter and are known graphically (Figures 1 and 2), so it is possible to solve the integration numerically.

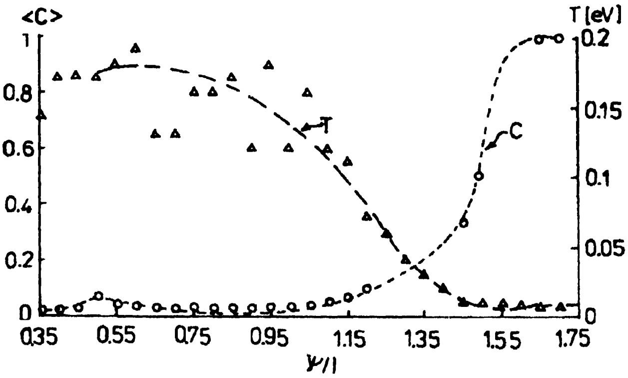

Figure 2. Instability factor (<C>) and temperature (T) as a functions of ψ/l

The experimental data of $<C>$ and T [2, 7] are for the region up to $\psi \cong 0.9$. We are thus led to conclude that (a) the instability grows in space up to $(C)(\psi) \cong 1$ since the instability condition remains valid in this region ; (b) the temperature gradient is negative so that we can extend the data to include $T(\psi_w) = 0$ (note Figures 1 and 2).

4. Results and conclusions

We consider the normalized densities ($N = n/n_c$). Figures 3 and 4 show the solutions of eqs. (16), (17) and (18) and the in comparisons with experimental results.

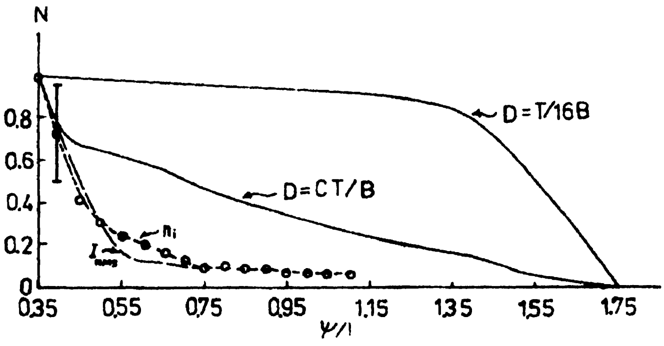

Figure 3. Normalized density (N) according to the solution of eqs (16) and (17) and comparison with experimental results

The experimental results [2] are of two types : either the result is for DC level of ion density, or is for a RMS of the fluctuated saturation current (both estimated by using an OML probe). These results depicted a sharp slope $(\partial n / \partial \psi)$ just after the critical surface. This means that there is a certain type of diffusion in this region (Bohm type).

It is obvious that the density profile for the case of $C = 1/16$ is too far from the behavior of the experimental one, where the normal form of D_B does not consider the fluctuation situation of the region [7]. In this case, $1/<B>T$ may control the distribution.

Taking into account of C as an instability factor, the profile gives a better agreement with the behavior of experimental curve. So not only $1/<B>T$ can control the distribution but the reciprocal of $<C>$ as well. The effect of $<C>$ shows an important role of the fluctuation in density distribution outside ψ_c (Figures 3 and 4). The sharp slope of the density can be referred to $<C>$ effect.

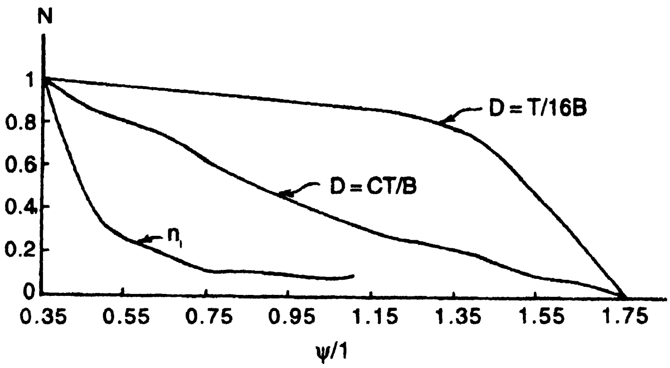

Figure 4. Normalized density (N) according to the solution of equation (18), and comparison with experimental results.

The discrepancy between the theoretical and experimental distributions may be due to some systematic error owing to many experimental complications, *e.g.* the difficulty in survey of a wide range of ψ in a single run of experiment owing to the limitation of probe manipulating system. In addition, the mentioned experiment has not been designed for this theory of $<C>$ factor. So we suggest further experimental investigations.

We did not find significant difference between the solutions of non-average (Figure 4) and average (averaging around the closed line) (Figure 3) results. That is because of the small variation of U-function $(\partial U / \partial \psi)$ in the region under investigation.

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References

[1] M G Rusbridge and E B Meservey *Phys. Fluids* **12** 1917 (1969)

[2] M I Sanduk *PhD Thesis* (Manchester, UK.) (1990)

[3] Dale M Meade *Phys. Rev. Lett.* **17** 677 (1966)

[4] T Ohkawa, M Yoshikawa, R E Kribel, A A Schupp and T H Jensen *Phys. Rev. Lett.* **24** 95 (1970)

[5] T Ohkawa, J R Gilleland, T Tammano, T Takeda and D K Bhadra *Phys. Rev. Lett.* **27** 1179 (1971)

[6] G D Hobbs and T B Taylor *Plasma Phys.* **10** 207 (1968)

[7] M I Sanduk *Indian J. Phys.* **71B** (4) 495 (1997)

[8] F F Chen *Introduction to Plasma and Controlled Fusion* (New York : Plenum) (1985)

[9] P S Carter, D N Edwards, P R Mossack and M G Rusbridge *Plasma Phys.* **23** 819 (1981)

[10] K Phillips, M G Rusbridge and K M Young *Plasma Phys.* **20** 653 (1978)